EXCEL IN MATHSCI

HIGHER MATHEMATICS | TOPIC PRACTICE LIBI

2026-27 | Original worksheets and separate worked answer booklets

Contents

03	**Functions and Graphs**	18 questions | worksheet + answers
04	**Vectors**	18 questions | worksheet + answers
05	**Quadratics and Polynomials**	18 questions | worksheet + answers
06	**Trigonometric Equations**	18 questions | worksheet + answers
07	**Differentiation**	18 questions | worksheet + answers
08	**Integration**	18 questions | worksheet + answers
09	**Straight Line**	18 questions | worksheet + answers
10	**Circle**	18 questions | worksheet + answers
11	**Recurrence Relations**	18 questions | worksheet + answers
12	**Optimisation and Area Between Curves**	18 questions | worksheet + answers

Suggested use

1. Use Section A to identify and repair prerequisite skill gaps.
2. Set Section B as topic-specific classwork or homework.
3. Use Section C for exam-style reasoning and assessment.
4. Mark with the separate worked answers; accept equivalent valid methods.
5. Do not give the answer booklets to pupils before an assessment.

All questions are newly written using the skill coverage of the supplied portfolios.
The resources are independently authored and are not official Qualifications Scotland papers.

© 2026 Excel in MathSci Ltd. All rights reserved.
Reproduction, distribution or resale requires prior written permission.

Tuition: 07577 482130 | info@excelinmathsci.co.uk | www.excelinmathsci.co.uk

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 3

Functions and Graphs

PUPIL WORKSHEET

Skills covered: Intercepts; completing the square; domains and ranges; composite and inverse functions; graph transformations; exponential and logarithmic graphs.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[3 marks]

Find the coordinates of the points where $y = x^2 - 5x + 6$ meets the coordinate axes.

Question 2

[3 marks]

Express $x^2 - 6x + 4$ in completed-square form and state its minimum value.

Question 3

[3 marks]

Express $1 + 4x - x^2$ in completed-square form and state its maximum turning point.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[2 marks]

State the range of $f(x) = 2(x+1)^2 - 3$, for $x \in \mathbb{R}$.

Question 5

[2 marks]

Let $A = \{-2, -1, 0, 1, 2\}$ and $B = \{0, 2, 4, 6\}$. Find $A \cap B$ and $A \cup B$.

Question 6

[3 marks]

State the largest possible domain in $\mathbb{R}$ of each function: (a) $f(x) = \sqrt{x-4}$; (b) $g(x) = \dfrac{x-2}{x^2-9}$.

SECTION B | Topic practice

Question 7

[4 marks]

Given $f(x) = 3x - 2$ and $g(x) = x^2 + 1$, find (a) $f(g(x))$ and (b) $g(f(x))$.

Question 8

[4 marks]

Given $f(x) = 4x + 7$, find $f^{-1}(x)$. Show that $f(f^{-1}(x)) = x$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 9

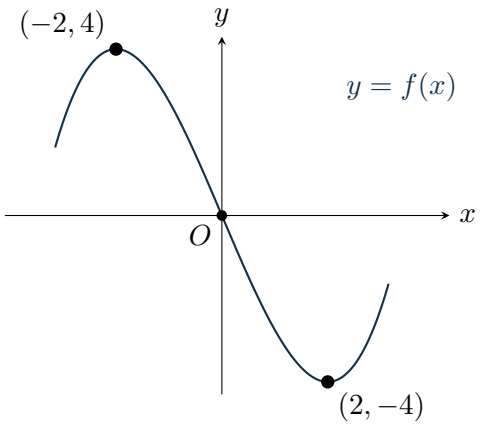
[4 marks]

Express $g(x) = 3x^2 - 12x + 8$ in the form $a(x+b)^2 + c$. Hence state the coordinates of its minimum turning point.

Question 10

[4 marks]

The diagram shows $y = f(x)$, with a maximum turning point at $(-2, 4)$ and a minimum turning point at $(2, -4)$. Sketch $y = f(x-1) + 2$, indicating the coordinates of both turning points.



Question 11

[5 marks]

For the function f in Question 10, sketch $y = -2f(x+1)$ and state the coordinates and natures of its turning points.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 12

[4 marks]

For $f(x) = x^2 - 4x + 1$, find simplified expressions for (a) $f(x+2)$, (b) $f(x) + 2$, (c) $f(2x)$.

Question 13

[4 marks]

For $y = 2^{x-1} + 3$, state the domain, range and equation of the horizontal asymptote. Give one point on the curve.

Question 14

[4 marks]

For $y = \log_3(x+2)$, state the vertical asymptote and domain. Find the points at which $y = 0$ and $y = 1$.

SECTION C | Exam-style and challenge

Question 15

[5 marks]

Given $f(x) = \dfrac{2x-5}{3}$, find $f^{-1}(x)$. Hence solve $f(x) = f^{-1}(x)$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 16

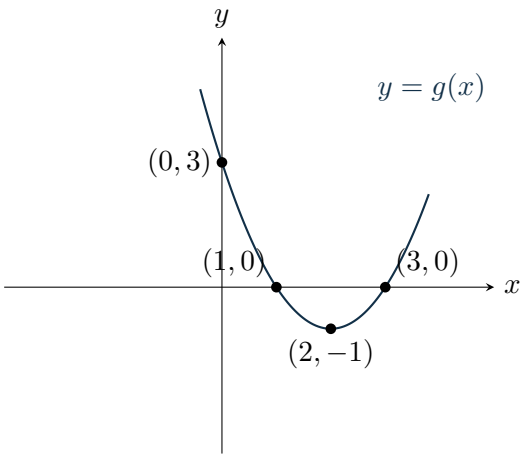
[5 marks]

The equation $x^2 + (k-2)x - k - 3 = 0$ has one root $x = -1$. Find k and hence the second root.

Question 17

[5 marks]

The diagram shows $y = g(x)$. Sketch $y = g(2 - x) + 1$, labelling the turning point and all intercepts.



Question 18

[5 marks]

Let $f(x) = \sqrt{x+1}$ and $g(x) = 2x - 5$. Find $f(g(x))$ and $g(f(x))$, stating the real domain of each.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 4

Vectors

PUPIL WORKSHEET

Skills covered: Two- and three-dimensional vectors; position vectors; magnitude; scalar product; collinearity; internal division; three-dimensional angles.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[3 marks]

Given $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$, express $2\mathbf{a} - \mathbf{b}$ in unit-vector form.

Question 2

[2 marks]

Find the magnitude of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

Question 3

[3 marks]

Given $A(1, -2, 3)$ and $B(5, 1, -1)$, find $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[2 marks]

Find $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = (3, -2, 1)$ and $\mathbf{b} = (2, 4, -1)$.

Question 5

[3 marks]

Find k if $(2, k, -1)$ is perpendicular to $(3, 2, 4)$.

Question 6

[2 marks]

The position vectors of P and Q are $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $-\mathbf{i} + 5\mathbf{j} + \mathbf{k}$. Find $\overrightarrow{PQ}$.

SECTION B | Topic practice

Question 7

[4 marks]

Show that the points $A(1, 2, 0)$, $B(3, 6, 2)$ and $C(6, 12, 5)$ are collinear.

Question 8

[4 marks]

The points $A(2, -1, 4)$ and $B(8, 5, -2)$ are joined by a line segment. Find the coordinates of the point P which divides AB internally in the ratio $AP : PB = 1 : 2$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 9 [3 marks]

Find the angle between $\mathbf{a} = (2, 1, 2)$ and $\mathbf{b} = (1, 2, -2)$.

Question 10 [4 marks]

Given $\mathbf{p} = (1, 2, 2)$ and $\mathbf{q} = (2, -1, 2)$, calculate the acute angle between the vectors, to one decimal place.

Question 11 [4 marks]

In a triangle, $A(1, 0, 2)$, $B(5, 2, 1)$ and $C(2, 4, 3)$. Find the coordinates of the midpoint M of BC and $\overrightarrow{AM}$.

Question 12 [3 marks]

Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. Determine whether the vectors are perpendicular.

Question 13 [3 marks]

For $A(1, 3, -2)$, $B(4, 1, 0)$ and $C(6, 5, 1)$, find $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 14

[3 marks]

Write $\mathbf{u} = (3, -4, 12)$ as a unit vector in the direction of $\mathbf{u}$.

SECTION C | Exam-style and challenge

Question 15

[5 marks]

For $A(0, 1, 2)$, $B(4, 3, 0)$ and $C(1, 5, 4)$, calculate $\angle BAC$ to one decimal place.

Question 16

[4 marks]

The points $A(2, 1, -1)$, $B(8, 7, 5)$, and $P(t, 4, 2)$ lie on the same straight line. Find t.

Question 17

[5 marks]

The points $O = (0, 0, 0)$, $A = (4, 0, 0)$, $B = (0, 3, 0)$, and $P = (2, 1, 6)$ are vertices of a tetrahedron. Find $\angle APB$ to one decimal place.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 18

[5 marks]

A point R divides the line segment from $P(-2, 4, 1)$ to $Q(7, -2, 10)$ in the ratio $PR : RQ = 2 : 1$. Find R. Hence calculate $|\overrightarrow{PR}|$.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 5

Quadratics and Polynomials

PUPIL WORKSHEET

Skills covered: Factorising; quadratic roots and the expression b-squared minus four ac; tangency; factor and remainder theorems; cubic intersections.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[2 marks]

Factorise fully $3x^2 - 11x - 4$.

Question 2

[3 marks]

Solve $2x^2 + 3x - 9 = 0$.

Question 3

[3 marks]

Determine the nature of the roots of $3x^2 - 6x + 5 = 0$. Give a reason.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[3 marks]

Find k if $x^2 + 6x + k = 0$ has equal roots.

Question 5

[2 marks]

Find the remainder when $P(x) = x^3 - 2x^2 - 5x + 6$ is divided by $x - 2$.

Question 6

[3 marks]

Factorise $x^3 - 4x$ fully and solve $x^3 - 4x = 0$.

SECTION B | Topic practice

Question 7

[5 marks]

Show that $x = -1$ is a root of $x^3 - 4x^2 + x + 6 = 0$. Hence solve the equation.

Question 8

[4 marks]

$P(x) = x^3 + ax^2 - 7x + 5$ has a factor $x - 1$. Find a and the remainder when $P(x)$ is divided by $x + 2$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 9 [5 marks]

The line $y = 2x + k$ is a tangent to $y = x^2 - 4x + 7$. Find k and the point of contact.

Question 10 [4 marks]

Find all real values of p for which $x^2 + (p+1)x + 2p = 0$ has equal roots.

Question 11 [4 marks]

Factorise $x^3 - 3x^2 - 4x + 12$ and state the x-intercepts of the corresponding cubic.

Question 12 [4 marks]

Find the coordinates of the points of intersection of $y = x^2 - 3x + 1$ and $y = x + 1$.

Question 13 [4 marks]

Find all the points where $y = 2x^3 - 5x^2 - 3x$ meets the x-axis.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 14 [3 marks]

The polynomial $P(x) = x^3 - 2x^2 + mx + 6$ has a factor $x + 1$. Find m.

SECTION C | Exam-style and challenge

Question 15 [5 marks]

Show that $x = 1$ is a root of $x^2 + (k+2)x - k - 3 = 0$ for every real k. Find k if the roots are equal.

Question 16 [5 marks]

Find the possible values of m for which $y = mx + 2$ is tangent to $y = x^2 - 4x + 6$.

Question 17 [4 marks]

Solve the equation $x^4 - 5x^2 + 4 = 0$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 18 [5 marks]

$P(x) = x^3 + kx^2 - 4x - 4k$. Factorise $P(x)$ fully and state the values of k for which $P(x) = 0$ has three distinct real roots.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 6

Trigonometric Equations

PUPIL WORKSHEET

Skills covered: Sine, cosine and tangent equations; degrees and radians; multiple angles; double-angle formulae; equations reducible to tangent; harmonic form.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[3 marks]

Solve $2\sin x^\circ = \sqrt{3}$ for $0 \leq x < 360$.

Question 2

[3 marks]

Solve $\cos 2x^\circ = 0$ for $0 \leq x < 180$.

Question 3

[3 marks]

Solve $\tan(x + 20)^\circ = 1$ for $0 \leq x < 360$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[2 marks]

Solve $\sin x^\circ = -\frac{1}{2}$ for $0 \leq x < 360$.

Question 5

[2 marks]

Solve $\cos x = \frac{1}{2}$ for $0 \leq x < 2\pi$.

Question 6

[2 marks]

Solve $\tan x = \sqrt{3}$ for $0 \leq x < \pi$.

SECTION B | Topic practice

Question 7

[4 marks]

Solve $2\sin 2x^\circ = \sqrt{3}$ for $0 \leq x < 360$.

Question 8

[4 marks]

Solve $3\cos 2x^\circ = -\frac{3}{2}$ for $0 \leq x < 360$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 9

[5 marks]

Solve $\sin 2x^\circ = \sin x^\circ$ for $0 \leq x < 360$.

Question 10

[4 marks]

Solve $2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ for $0 \leq x < 360$.

Question 11

[5 marks]

Solve $\cos 2x^\circ = \sin x^\circ$ for $0 \leq x < 360$.

Question 12

[4 marks]

Solve $3\sin x^\circ = 4\cos x^\circ$ for $0 \leq x < 360$. Give answers to one decimal place.

Question 13

[4 marks]

Solve $\sin\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ for $0 \leq x < 2\pi$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 14

[6 marks]

Express $3\sin x^\circ + 4\cos x^\circ$ in the form $R\sin(x+\alpha)^\circ$, with $R > 0$ and $0 < \alpha < 90$. Hence solve $3\sin x^\circ + 4\cos x^\circ = 0$ for $0 \leq x < 360$.

SECTION C | Exam-style and challenge

Question 15

[4 marks]

Solve $2\sin^2 x^\circ + \sin x^\circ - 1 = 0$ for $0 \leq x < 360$.

Question 16

[5 marks]

Solve $2\cos 2x^\circ + 3\cos x^\circ = 0$ for $0 \leq x < 360$. Give answers to one decimal place.

Question 17

[4 marks]

Solve $\sin 2x^\circ = 2\cos x^\circ$ for $0 \leq x < 360$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 18 [6 marks]

Express $4\sin x^\circ + 3\cos x^\circ$ in the form $R\sin(x+\alpha)^\circ$. Hence solve $4\sin x^\circ + 3\cos x^\circ = 2$, for $0 \leq x < 360$. Give answers to one decimal place.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 7

Differentiation

PUPIL WORKSHEET

Skills covered: Indices and surds; differentiating algebraic and trigonometric functions; chain rule; tangents; stationary points and their nature; rates of change.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[3 marks]

Differentiate $y = x^4 - 3x^2 + \dfrac{2}{x}$ with respect to x.

Question 2

[4 marks]

Express $y = 4\sqrt{x^3} - \dfrac{3}{x^2}$ using indices. Hence find $\dfrac{dy}{dx}$.

Question 3

[2 marks]

Find the derivative of $y = (x + 3)(x - 4)$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[2 marks]

Find $f'(x)$ for $f(x) = \dfrac{x^2 + 2x}{x}$, $x \neq 0$.

Question 5

[2 marks]

Differentiate $y = 5\sin x - 3\cos x$ with respect to x, where x is in radians.

Question 6

[3 marks]

Differentiate $f(x) = (2x + 1)^5$.

SECTION B | Topic practice

Question 7

[4 marks]

Given $f(x) = \sqrt{3x + 1}$, find $f'(x)$ and evaluate $f'(1)$.

Question 8

[3 marks]

Differentiate $y = (x^2 - 4x + 1)^3$ with respect to x.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 9

[4 marks]

Find the equation of the tangent to $y = x^2 - 3x + 5$ at the point where $x = 2$.

Question 10

[6 marks]

Find the coordinates of the stationary points of $f(x) = x^3 - 6x^2 + 9x$ and determine their nature using the second derivative test.

Question 11

[6 marks]

For $f(x) = x^3 - 3x^2 - 9x + 2$, find the coordinates and natures of both stationary points.

Question 12

[5 marks]

Find the intervals on which $f(x) = x^3 - 6x^2 + 9x + 2$ is strictly increasing and strictly decreasing.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 13

[3 marks]

Differentiate $y = 3\cos 2x$, where x is in radians.

Question 14

[5 marks]

The displacement s metres of a particle after t seconds is $s(t) = t^3 - 6t^2 + 8t$. Find its velocity and acceleration at $t = 2$.

SECTION C | Exam-style and challenge

Question 15

[5 marks]

Find the equation of the tangent to $y = x^3 - 3x^2 - 9x + 5$ at $x = 1$. Give the equation in the form $ax + by = c$, with integer coefficients.

Question 16

[6 marks]

$f(x) = x + \dfrac{1}{x}$, $x \neq 0$. Find its stationary points and state their nature.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 17

[5 marks]

The curve has equation $f(x) = x^3 - 3x$. Sketch the graph of $y = f'(x)$, labelling its x-intercepts and minimum turning point.

Question 18

[6 marks]

For $f(x) = x^4 - 8x^2 + 9$, determine the coordinates and natures of all stationary points.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 8

Integration

PUPIL WORKSHEET

Skills covered: Indefinite integrals; fractional and negative powers; standard composite-function integrals; trigonometric integrals; definite integrals; initial conditions.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[3 marks]

Find $\int (6x^2 - 4x + 3)\,dx$.

Question 2

[3 marks]

Find $\int 5\sqrt{x}\,dx$, where $x > 0$.

Question 3

[3 marks]

Find $\int \left(\frac{4}{x^3} + x^2\right)\,dx$, where $x \neq 0$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[2 marks]

Find $\int 3\sin x\,dx$, where x is in radians.

Question 5

[2 marks]

Find $\int 4\cos x\,dx$, where x is in radians.

Question 6

[3 marks]

Find $\int (2x-3)^4\,dx$.

SECTION B | Topic practice

Question 7

[3 marks]

Evaluate $\int_0^2 (3x^2+2x)\,dx$.

Question 8

[3 marks]

Evaluate $\int_1^3 (6x-4)\,dx$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 9

[3 marks]

Find $\int \cos 3x \, dx$.

Question 10

[3 marks]

Find $\int \sin\left(2x + \frac{\pi}{3}\right) \, dx$.

Question 11

[4 marks]

Given $f'(x) = 6x^2 - 4x$ and $f(1) = 5$, find an expression for $f(x)$.

Question 12

[4 marks]

A curve satisfies $\frac{dy}{dx} = 3x^2 - 12x + 5$ and passes through $(2, 7)$. Find its equation.

Question 13

[4 marks]

Evaluate $\int_0^1 (2x + 1)^3 \, dx$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 14

[3 marks]

Given $\int_0^a 2x\,dx = 25$, where $a > 0$, find a.

SECTION C | Exam-style and challenge

Question 15

[4 marks]

Given $\int_1^a 3x^2\,dx = 7$, where $a > 1$, find a.

Question 16

[4 marks]

Find $\int \left(\sqrt{x} - \frac{2}{\sqrt{x}}\right) dx$, where $x > 0$.

Question 17

[6 marks]

Given $f''(x) = 12x - 6$, $f'(0) = 4$ and $f(0) = -1$, find $f(x)$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 18

[4 marks]

Evaluate exactly $\int_0^{\frac{\pi}{2}} (2\sin x + \cos x)\, dx$.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 9

Straight Line

PUPIL WORKSHEET

Skills covered: Gradients and equations; parallel and perpendicular lines; angle of inclination; midpoints and distances; medians, altitudes and perpendicular bisectors.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[2 marks]

Find the gradient of the line passing through $A(-2, 5)$ and $B(4, -1)$.

Question 2

[3 marks]

Find the equation of the line with gradient $\frac{3}{2}$ passing through $(2, -1)$. Give your answer in the form $ax + by = c$.

Question 3

[3 marks]

Find the equation of the line through $(-3, 4)$ parallel to $2x + 3y = 12$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4 [3 marks]

Find the equation of the line through $(4, 1)$ perpendicular to $y = -4x + 5$.

Question 5 [2 marks]

Find the coordinates of the midpoint of the line segment joining $(-3, 2)$ and $(5, -4)$.

Question 6 [2 marks]

Calculate the distance between $(1, 2)$ and $(7, 10)$.

SECTION B | Topic practice

Question 7 [4 marks]

A line has an angle of inclination of $135°$ and passes through $(2, 5)$. Find its equation in the form $ax + by = c$.

Question 8 [4 marks]

Find the coordinates of the point of intersection of $3x + 2y = 11$ and $x - y = 2$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 9

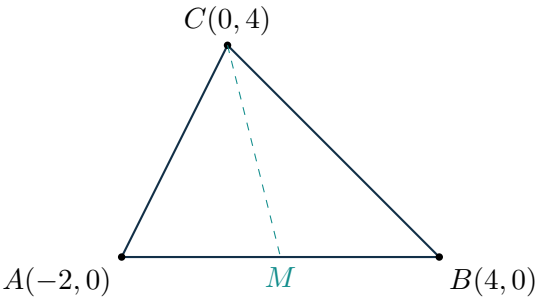
[4 marks]

Find the equation of the line through $P(3, -2)$, perpendicular to $5x - 2y = 8$.

Question 10

[5 marks]

Triangle ABC has vertices $A(-2, 0)$, $B(4, 0)$ and $C(0, 4)$. Find the equation of the median from C to AB.



Question 11

[5 marks]

Find the equation of the perpendicular bisector of the line segment joining $A(-4, 1)$ to $B(2, 7)$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 12

[3 marks]

Calculate the angle of inclination of the line $2x - 3y = 6$, giving your answer to one decimal place.

Question 13

[4 marks]

Find the perpendicular distance between the parallel lines $y = 2x - 3$ and $y = 2x + 7$.

Question 14

[5 marks]

$A(2, -1)$, $B(-4, 3)$ and $C(6, 5)$ are the vertices of a triangle. Find the equation of the median from A.

SECTION C | Exam-style and challenge

Question 15

[6 marks]

Triangle ABC has vertices $A(1, 1)$, $B(7, 3)$ and $C(3, 9)$. Find the equations of the altitude and median from C.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 16

[3 marks]

A line has equation $3x - 2y = 12$. Find its angle of inclination, to one decimal place.

Question 17

[5 marks]

The lines $y = (k-1)x + 2$ and $2x + (k+3)y = 5$ are perpendicular. Find k.

Question 18

[6 marks]

$ABCD$ is a parallelogram with $A(0,0)$, $B(6,2)$ and $D(-2,6)$. Show that $\angle BAD = 90°$, find C, and determine the equation of diagonal BD.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 10

Circle

PUPIL WORKSHEET

Skills covered: Circle equations; completing the square; diameter; point location; equations with no real circle; line–circle intersections; tangent equations; circle geometry.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[2 marks]

Find the equation of the circle with centre $(-2, 3)$ and radius 5.

Question 2

[4 marks]

Find the centre and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.

Question 3

[4 marks]

$A(1, -2)$ and $B(7, 6)$ are the endpoints of a diameter. Find the equation of the circle.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[3 marks]

For the circle $(x-1)^2+(y-2)^2=9$, state whether $P(4,2)$ and $Q(1,7)$ are inside, on, or outside the circle.

Question 5

[3 marks]

Explain why $x^2+y^2+4x-2y+10=0$ does not represent a real circle.

Question 6

[2 marks]

Find the points at which $x^2+y^2=16$ intersects the x-axis.

SECTION B | Topic practice

Question 7

[3 marks]

Find the points of intersection of $(x-1)^2+(y+2)^2=20$ and the line $y=0$.

Question 8

[4 marks]

A circle has equation $x^2+y^2-4x-6y+9=0$. Find its points of intersection with $y=3$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 9

[4 marks]

Find the equation of the tangent to $x^2 + y^2 = 25$ at $P(3, 4)$.

Question 10

[4 marks]

The line $y = mx + 5$ is tangent to $x^2 + y^2 = 25$. Find m.

Question 11

[5 marks]

Two identical circles with centres $C_1(1, 2)$ and $C_2(7, 2)$ touch externally. Find their radius, point of contact, and both equations.

Question 12

[5 marks]

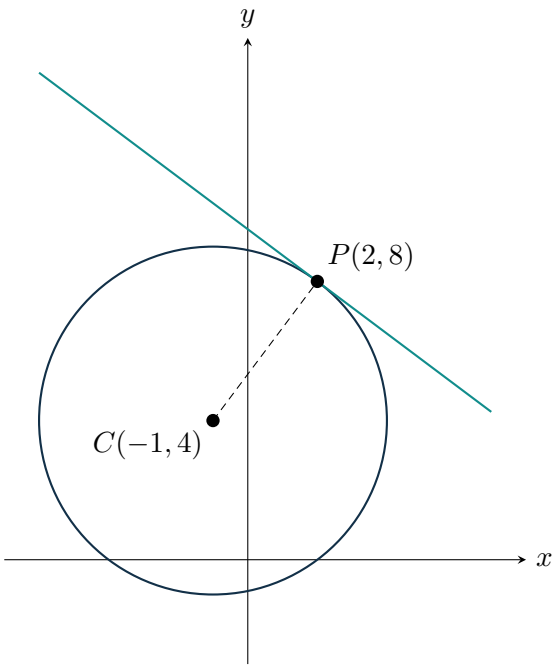
Find the values of k for which $y = x + k$ is tangent to $(x - 1)^2 + (y + 2)^2 = 8$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 13

[4 marks]

The diagram shows a circle with centre $C(-1, 4)$ and the point $P(2, 8)$ on its circumference. Find the equation of the tangent at P.



Question 14

[5 marks]

Find the exact coordinates of the points where the line $y = 2x + 5$ meets the circle $(x + 2)^2 + (y - 1)^2 = 25$.

SECTION C | Exam-style and challenge

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 15 [5 marks]

The circle $x^2 + y^2 - 2x + 4y + k = 0$ is tangent to the line $y = 3$. Find k.

Question 16 [6 marks]

Find the exact points of intersection of $(x-1)^2 + (y-1)^2 = 10$ and $(x-5)^2 + (y-1)^2 = 10$.

Question 17 [5 marks]

A circle passes through $A(0,0)$, $B(4,0)$ and $C(0,6)$. Find its equation.

Question 18 [6 marks]

From $P(8,0)$, two tangents are drawn to $x^2 + y^2 = 16$. Find their points of contact and equations.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 11

Recurrence Relations

PUPIL WORKSHEET

Skills covered: Percentage change; linear recurrence formulae; computing terms; fixed limiting values; finding coefficients; financial and population models.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[3 marks]

An investment of $£2500$ increases by 4% each year. Calculate its value after 3 years.

Question 2

[3 marks]

A machine valued at $£1200$ loses 8% of its value each year. Find its value after 4 years, to the nearest penny.

Question 3

[3 marks]

Given $u_0 = 5$ and $u_{n+1} = 2u_n - 3$, calculate u_1, u_2, u_3.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[3 marks]

Given $u_0 = 4$ and $u_{n+1} = \frac{1}{2}u_n + 6$, calculate u_1, u_2, u_3.

Question 5

[4 marks]

The first three terms of a linear recurrence relation $u_{n+1} = au_n + b$ are $6, 10, 18$. Find a and b.

Question 6

[3 marks]

The recurrence $u_{n+1} = 0.75u_n + 12$ approaches a limiting value L. Find L.

SECTION B | Topic practice

Question 7

[5 marks]

A savings account contains $£18000$. Interest of 3% is added each year before $£1000$ is withdrawn. Write down a recurrence relation for the balance u_n, and find the balance after 3 years.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 8

[4 marks]

A patient is given 15 mg of a drug at the end of every day. During each day 40% of the drug already present is removed. Initially there is no drug in the bloodstream. Form a recurrence relation and calculate the amount immediately after the third dose.

Question 9

[5 marks]

A loan of $£4000$ is charged interest at 2% per month. A repayment of $£350$ is made at the end of every month. Write down a recurrence and find the amount outstanding after 3 payments.

Question 10

[4 marks]

A wildlife reserve contains 700 rabbits. Each month 30% of those present leave and 120 new rabbits arrive. Write down a recurrence relation and find its limiting population.

Question 11

[4 marks]

Given $u_0 = 10$ and $u_{n+1} = 0.4u_n + 9$, calculate u_1, u_2 and find the limiting value.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 12

[4 marks]

Three consecutive terms of a linear recurrence $u_{n+1} = au_n + b$ are $4, 7, 13$. Determine a and b.

Question 13

[3 marks]

The recurrence $u_{n+1} = ku_n + 8$ has a limiting value of 40. Find k.

Question 14

[4 marks]

A recurrence is defined by $u_0 = 1000$ and $u_{n+1} = 0.9u_n + 50$. Find the limiting value. State whether the first three terms are above or below it.

SECTION C | Exam-style and challenge

Question 15

[4 marks]

The first three terms of a linear recurrence are $3, 8, 18$. Find the recurrence in the form $u_{n+1} = au_n + b$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 16

[5 marks]

A club has 200 members. At the start of each year 75% renew their membership and 60 new people join. Model the total just after enrolment with a recurrence relation and determine whether the membership will ever reach 250, assuming these rates remain constant.

Question 17

[5 marks]

Given $u_0 = 75$ and $u_{n+1} = 0.6u_n + 18$, find the first value of n for which $u_n < 50$.

Question 18

[6 marks]

A debt of $£1200$ attracts 5% interest monthly. A payment of $£200$ is made after the interest is added. Find the number of monthly payments needed to clear the debt, assuming a smaller final payment is allowed.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 12

Optimisation and Area Between Curves

PUPIL WORKSHEET

Skills covered: Maximum and minimum values; optimisation in geometric and practical contexts; definite integrals for area; signed regions and areas between curves.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[4 marks]

Find the maximum value of $f(x) = -x^2 + 6x + 2$, and the value of x at which it occurs.

Question 2

[3 marks]

Evaluate $\int_0^3 (4x - x^2)\,dx$.

Question 3

[5 marks]

Find the total area bounded by $y = x^2 - 4x$ and the x-axis.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 4

[2 marks]

Find the x-coordinates of the intersections of $y = 4x$ and $y = x^2$.

Question 5

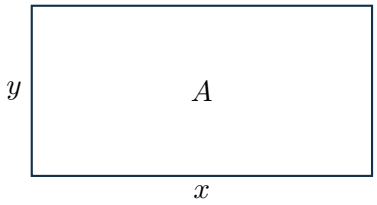
[4 marks]

Find the stationary point of $y = 3x^2 - 12x + 5$ and state its nature.

Question 6

[4 marks]

A rectangle has perimeter 20 cm. Its sides are x cm and y cm. Show that its area is $A(x) = 10x - x^2$, and state the permitted values of x.



SECTION B | Topic practice

Question 7

[4 marks]

For the rectangle in question 6, find the maximum possible area and the dimensions of the rectangle.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 8

[6 marks]

An open box is made from a 20 cm by 12 cm card by cutting equal squares of side x cm from the corners and folding the sides. (a) Show that $V = x(20 - 2x)(12 - 2x)$. (b) Find the value of x giving maximum volume, to two decimal places.

Question 9

[4 marks]

The height of a ball, h metres, after t seconds is given by $h(t) = 12t - t^2$, $0 \le t \le 12$. Find the greatest height reached and the time taken to reach it.

Question 10

[5 marks]

Fencing of length 30 m is used to enclose three sides of a rectangular garden against a straight wall. Find the dimensions that give the greatest area.

Question 11

[4 marks]

Calculate the area enclosed by $y = 6x - x^2$ and the x-axis.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 12

[5 marks]

Find the exact area enclosed by $y = 6$ and $y = x^2$.

Question 13

[5 marks]

Find the area enclosed between the line $y = 2x + 3$ and the parabola $y = x^2$.

Question 14

[5 marks]

Find the total area enclosed between $y = x^3 - 4x$ and the x-axis, for $-2 \leq x \leq 2$.

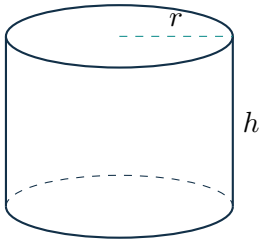
SECTION C | Exam-style and challenge

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 15

[6 marks]

A closed cylindrical container has a fixed volume of $250\pi\,\text{cm}^3$. If its radius is r cm and height is h cm, show that its surface area is $S = 2\pi r^2 + \dfrac{500\pi}{r}$. Find the dimensions giving minimum surface area.



Question 16

[5 marks]

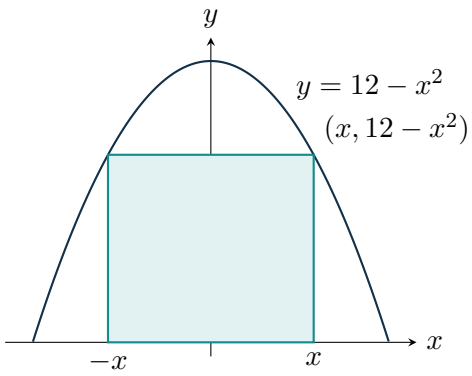
Find the exact area enclosed between $y = 4 - x^2$ and $y = x + 2$.

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.

Question 17

[6 marks]

A rectangle is placed symmetrically beneath the curve $y = 12 - x^2$, with one side on the x-axis and its two upper vertices on the curve. Find its maximum possible area.



Question 18

[6 marks]

Find the total area enclosed between the cubic $y = x^3 - 4x$ and the line $y = 2x$.

— End of pupil worksheet —

© 2026 Excel in MathSci Ltd. All rights reserved. Reproduction or resale requires permission.