EXCEL IN MATHSCI
HIGHER MATHEMATICS • TOPIC RESOURCE

2026–27
Portfolio 12

Optimisation and Area Between Curves

PUPIL WORKSHEET

Skills covered: Maximum and minimum values; optimisation in geometric and practical contexts; definite integrals for area; signed regions and areas between curves.

Instructions: Attempt all questions. Show sufficient working to justify your answers. Give exact values unless a degree of accuracy is stated. Each section is separate for targeted revision.

Prerequisite core skills revise earlier learning; *topic practice* develops individual techniques; *exam-style and challenge* combines skills.

Pupil: _____ **Date:** _____

SECTION A | Prerequisite core skills

Question 1

[4 marks]

Find the maximum value of $f(x) = -x^2 + 6x + 2$, and the value of x at which it occurs.

Question 2

[3 marks]

Evaluate $\int_0^3 (4x - x^2)\,dx$.

Question 3

[5 marks]

Find the total area bounded by $y = x^2 - 4x$ and the x-axis.

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Question 4

[2 marks]

Find the x-coordinates of the intersections of $y = 4x$ and $y = x^2$.

Question 5

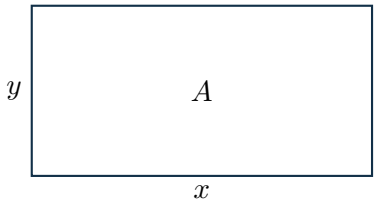
[4 marks]

Find the stationary point of $y = 3x^2 - 12x + 5$ and state its nature.

Question 6

[4 marks]

A rectangle has perimeter 20 cm. Its sides are x cm and y cm. Show that its area is $A(x) = 10x - x^2$, and state the permitted values of x.



SECTION B | Topic practice

Question 7

[4 marks]

For the rectangle in question 6, find the maximum possible area and the dimensions of the rectangle.

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Question 8

[6 marks]

An open box is made from a 20 cm by 12 cm card by cutting equal squares of side x cm from the corners and folding the sides. (a) Show that $V = x(20 - 2x)(12 - 2x)$. (b) Find the value of x giving maximum volume, to two decimal places.

Question 9

[4 marks]

The height of a ball, h metres, after t seconds is given by $h(t) = 12t - t^2$, $0 \le t \le 12$. Find the greatest height reached and the time taken to reach it.

Question 10

[5 marks]

Fencing of length 30 m is used to enclose three sides of a rectangular garden against a straight wall. Find the dimensions that give the greatest area.

Question 11

[4 marks]

Calculate the area enclosed by $y = 6x - x^2$ and the x-axis.

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Question 12

[5 marks]

Find the exact area enclosed by $y = 6$ and $y = x^2$.

Question 13

[5 marks]

Find the area enclosed between the line $y = 2x + 3$ and the parabola $y = x^2$.

Question 14

[5 marks]

Find the total area enclosed between $y = x^3 - 4x$ and the x-axis, for $-2 \leq x \leq 2$.

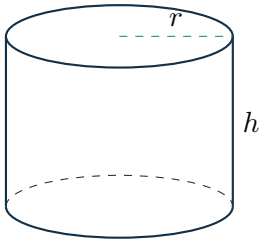
SECTION C | Exam-style and challenge

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Question 15

[6 marks]

A closed cylindrical container has a fixed volume of $250\pi\,\text{cm}^3$. If its radius is r cm and height is h cm, show that its surface area is $S = 2\pi r^2 + \dfrac{500\pi}{r}$. Find the dimensions giving minimum surface area.



Question 16

[5 marks]

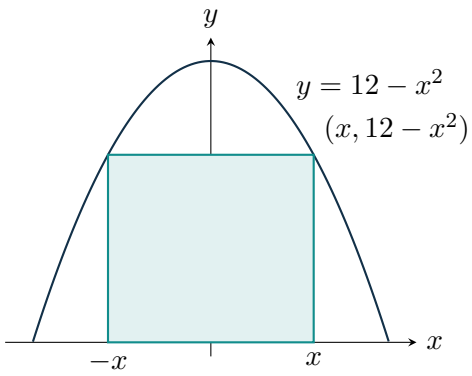
Find the exact area enclosed between $y = 4 - x^2$ and $y = x + 2$.

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Question 17

[6 marks]

A rectangle is placed symmetrically beneath the curve $y = 12 - x^2$, with one side on the x-axis and its two upper vertices on the curve. Find its maximum possible area.



Question 18

[6 marks]

Find the total area enclosed between the cubic $y = x^3 - 4x$ and the line $y = 2x$.

— End of pupil worksheet —

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