AH
National
Qualifications
2026

X847/77/11

Mathematics
Paper 1 (Non-calculator)

THURSDAY, 7 MAY

9:00 AM – 10:00 AM

Total marks — 35

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

You must leave your answer booklet on your desk; if you do not, you could lose all the marks for this paper.

Qualifications Scotland
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FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n=\frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n=\frac{a\left(1-r^n\right)}{1-r},\ r\neq 1$$

$$\sum_{r=1}^{n} r=\frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2=\frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3=\frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n=\sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \text{ where } \binom{n}{r}={}^nC_r=\frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x)=f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\frac{f^{iv}(0)x^4}{4!}+\ldots$$

De Moivre's theorem

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

MARKS

Total marks — 35

Attempt ALL questions

1. Differentiate:

(a) $y = 3x^4 \sec 2x$ **2**

(b) $f(x) = \dfrac{e^{5x}}{2x+1}$, simplifying your answer. **3**

2. A system of equations is given by

$$x + y - z = 9$$
$$2x - y + 3z = -2$$
$$3x + 2y - 2z = 21$$

Use Gaussian elimination to solve this system of equations. **4**

3. A complex number is defined by $z = \sqrt{3} + i$.

(a) Express z in polar form. **2**

(b) Use de Moivre's theorem to show that z^3 is purely imaginary. **2**

4. Find the particular solution of the differential equation

$$2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$$

given that $y = 2$ and $\dfrac{dy}{dx} = -1$ when $x = 0$. **5**

MARKS

5. Matrix A is defined by $A = \begin{pmatrix} 3 & 5 \\ -2 & x \end{pmatrix}$.

(a) State an expression for the determinant of A in terms of x. 1

Matrix A is multiplied by matrix B such that $\det AB = 12x + 40$.

(b) State the determinant of B. 1

The inverse of matrix B is $B^{-1} = \begin{pmatrix} 1 & -1 \\ -\frac{5}{4} & \frac{3}{2} \end{pmatrix}$.

(c) Find matrix B. 2

6. (a) Use the substitution $u = x - 1$ to find $\int x(x-1)^4 \, dx$. 3

(b) Hence find the exact volume of the solid formed by rotating the curve with equation $y = 2\sqrt{x}(x-1)^2$ about the x axis through 2π radians, from $x = 0$ to $x = 1$. 4

7. The complex number $z = 2 + i$ is a root of the polynomial equation $z^4 - 2z^3 - z^2 + 2z + 10 = 0$.

(a) State a second root of the equation. 1

(b) Find the remaining roots. 5

[END OF QUESTION PAPER]

AH National Qualifications 2026

X847/77/12

Mathematics Paper 2

THURSDAY, 7 MAY

10:30 AM – 1:00 PM

Total marks — 80

Attempt ALL questions.

You may use a calculator.

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MARKS

Total marks — 80

Attempt ALL questions

1. Differentiate $f(x) = 3\sin^{-1} 7x$. 2

2. Write down the binomial expansion of $\left(x^2 - \frac{5}{x}\right)^4$ and simplify your answer. 4

3. (a) Find and simplify the Maclaurin expansion, up to and including the term in x^3, for:

(i) e^{3x} 2

(ii) $\ln(1+x)$. 2

(b) Hence find and simplify the Maclaurin expansion, up to and including the term in x^3, for

$$e^{3x} \ln\left(\frac{1}{1+x}\right).$$ 2

4. (a) Use the Euclidean algorithm to find d, the greatest common divisor of 1428 and 567. 1

(b) Find integers a and b such that $1428a + 567b = d$. 2

5. Given $y = x^{4x}$, use logarithmic differentiation to find $\frac{dy}{dx}$.
Write your answer in terms of x. 5

MARKS

6. (a) An arithmetic sequence has terms $u_3 = 6$ and $u_{11} = 10$.

For this sequence, find the:

(i) common difference 1

(ii) first term 1

(iii) sum of the first 109 terms. 1

(b) The terms $v_3 = 18$ and $v_4 = 27$ form part of a geometric sequence.

For this sequence, find:

(i) the common ratio 1

(ii) the first term 1

(iii) an expression, in terms of n, for the sum of the first n terms. 1

(c) Find **algebraically** the least value of n such that the sum of the geometric series exceeds the sum of the first 109 terms in the arithmetic sequence. 1

7. A curve is defined parametrically by

$$x = 3\ln(2t+1),\ y = t - \frac{1}{2}t^2, \text{ where } t > 0.$$

(a) Find an expression for $\frac{dy}{dx}$. Simplify your answer. 2

(b) Find the coordinates of the stationary point on the curve. 2

8. The volume, V cubic metres, of water held in a reservoir is given by

$$V = 18\left(2+\sqrt{h}\right)^6 - 1152$$

where h metres is the depth of water.

Water is pumped out of the reservoir at a constant rate of 0.6 $m^3\,s^{-1}$.

Find the rate of change of the depth of water in the reservoir when the depth is 9 metres. 4

[Turn over

MARKS

9. Express 3442_5 in base 9. 3

10. A curve is defined by the equation $x^2e^{6y} + x^2 + y^5 = 50$.

(a) Find $\frac{dy}{dx}$ in terms of x and y. 4

(b) Given $x > 0$, explain why the derivative is never zero. 1

11. Find $\int \frac{2x+4}{x^2+4}\,dx$. 4

12. Prove by induction that $7^n + 2$ is divisible by 3 for all $n \in \mathbb{N}$. 5

13. (a) Express using partial fractions $\frac{x+3}{(x+7)(x+5)}$. 2

(b) Hence find the particular solution of the differential equation

$$\frac{dy}{dx} + \frac{2}{x+3}y = \frac{1}{(x+7)(x+5)(x+3)}, \text{ where } x \geq 0,$$

given that $y = 0$ when $x = 3$. 7

14. The plane π contains the points P$(2, 3, -4)$, Q$(3, 5, 1)$ and R$(6, 0, -6)$.

(a) Determine the Cartesian equation of π. 4

The line L has symmetric equations $\frac{x-8}{2} = \frac{y+2}{-1} = \frac{z+2}{3}$.

L intersects π at the point S.

(b) Find the coordinates of S. 3

(c) Calculate the size of the acute angle between L and π. 3

MARKS

15. Let r be a positive real number and consider the following statement:

If r is irrational then $\sqrt{r}$ is irrational.

(a) Write down the contrapositive of the statement. **1**

(b) Hence prove that the statement is true. **3**

16. (a) Given $y = \ln(\cos x)$, $0 \leq x < \frac{\pi}{2}$, show that $\frac{dy}{dx} = -\tan x$. **2**

For a function $g(x)$, it is known that

$$\int xg(x)\,dx = 2x\tan 2x - \int 2\tan 2x\,dx.$$

(b) (i) Determine the exact value of $\int_0^{\frac{\pi}{6}} xg(x)\,dx$. **2**

(ii) Find an expression for $g(x)$ in terms of x. **1**

[END OF QUESTION PAPER]